MAT 1348 3X – Practice Test # 3 – Spring/Summer 2016

Name:________________________________

Student Number:________________________________

MAT 1348 3X – Practice Test # 2 – Spring Summer 2016

Name:______________________________

Student Number:______________________________

Question	Possible Points	Points Obtained
# 1	5	
# 2	5	
# 3	5	
# 4	5	
# 5	5	
Total	30	

Instructions:

- Print your name and student number on the first two pages.
- Verify that your copy of the test has all of its 8 pages.
- You must answer all questions. There are 5 questions for a total of 25 points.
- Write the solutions to the questions in the space provided. You may use the back of the pages if necessary.

SHOW ALL YOUR WORK

1. (**5 pts**) For each of the following functions, indicate whether it is an injection, a surjection, and/or a bijection.

   (a) $f : \{a, b, c\} \to \{p, q\}$; $f(a) = f(b) = p, f(c) = q$

   (b) $g : \mathbb{N} \to \mathbb{N}$; $h(x) = x + 3$

   (c) $h : \mathbb{Z} \to \mathbb{Z}$; $l(x) = x - 3$

   (d) $k : \mathbb{N} \to \mathbb{R}$; $m(x) = 2x^2 - 4$

   (e) $l : \mathbb{R} \to \mathbb{R}$; $p(x) = -x^3 - 1$

2. (**5 pts**) Fo each of the following relations indicate whether it is reflexive, symmetric, transitive, and/or antisymmetric.

   (a) The relation R_1 on $\{a, b, c\}$ given by $R_1 = \{(a, b), (b, c), (a, c)\}$.

   (b) The relation R_2 on $\{a, b, c\}$ given by $R_2 = \{(a, a), (b, c), (c, b)\}$.

   (c) The relation R_3 on $\{a, b, c\}$ given by $R_3 = \{(a, a), (b, b), (c, c), (a, c)\}$.

   (d) The relation R_4 on $\mathbb{Z}$ given by $R_4 = \{(a, b) \mid a < b\}$.

   (e) The relation R_5 on $\mathbb{Z}$ given by $R_5 = \{(a, b) \mid b - a \leq 5\}$.

3. (**5 pts**) How many positive integers having exactly 4 numbers (i.e. the numbers between 1000 and 9999) are

   (a) divisible by 9?

   (b) have different numbers?

   (c) are divisible by 5 or by 7?

4. (**5 pts**) For each of the following statements indicate whether it is true or false.

(a) If R is an equivalence relation on a set X, then R is never anti-symmetric.

(b) The functions $f : A \to B$ and $g : B \to A$ are inverses if and only if $(f \circ g)(b) = b$ for all $b \in B$.

(c) Given R an equivalence relation on the non-empty set A: the set of distinct equivalence classes form a partition of A.

(d) A partition of the set A is a collection of subsets A_1, A_2, $\ldots$, A_n of A such that $A = A_1 \cup A_2 \cup \ldots \cup A_n$ and $A_i \cap A_j = \emptyset$ for $i \neq j$.

(e) If $f : A \to B$ is an injective function then f has an inverse.

5. (**5 pts**) Consider the function $\lceil - \rceil : \mathbb{R} \to \mathbb{Z}$ defined by

$$\lceil x \rceil = \text{ the smallest integer greater than or equal to } x.$$

(So, for example, $\lceil 4.21 \rceil = 5$, $\lceil -3.3 \rceil = -3$, $\lceil 12 \rceil = 12$, etcetera.)

(a) (**2 pts**) Prove that this function is surjective but not injective.

(b) (**2 pts**) Define a relation on $\mathbb{R}$ such that

$$(x, y) \in R \text{ if and only if } \lceil x \rceil = \lceil y \rceil.$$

Prove that this is an equivalence relation.

(c) (**1 pt**) Determine the equivalence class for the integers 1 and -1.